H2 Mathematics
Pure Mathematics Section
A Comprehensive Checklist of Skills and Concepts
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Syllabus topic 1: Functions and graphs

1.1 Checklist on Functions

- Determine whether composite/inverse function exists.
- Find composite/inverse function.
- Find restricted domain for composite/inverse function to exist.
- Find domain/range for a function.
- Sketch graphs of functions, their inverses and the line of reflection.
- Find the point of intersection between the function and its inverse, using the fact that it occurs on the line y = x.

1.2 Checklist on Graphing techniques

- Sketch basic graphs of parabolas, cubic/quartic functions, rectangular hyperbolas, hyperbolas, ellipses, circles and other standard functions (trigonometric, exponential and logarithmic).
- Sketch graphs of rational functions with emphasis on skills like:

1. Find axial intercepts.
2. Find equations of asymptotes.
3. Find stationary points.
4. Find range of values for which curve does not lie.
5. Find conditions for curve to have stationary values.
6. Find unknown variables given certain properties of curve.
7. Find the number of roots for a given equation, requiring one to insert an additional graph and finding the number of intersections between both.
8. Find an inequality, requiring one to investigate the slopes of asymptotes.

- Describe linear transformations (of reflections, translations, scalings) in words.

- Sketch curves after being transformed by reflections, translations, scalings or their combinations.
- Sketch graphs of y = f'(x) and appreciate concepts like concave up/down.
- Sketch graphs of the forms y = 1 / f(x), y^2 = f(x), y = f(|x|), y = |f(x)| with combinations of reflections, translations, scalings.
- Given transformed graph(s), sketch the original y = f(x); e.g. given y = 2 + f(2x + 1), sketch y = f(x).

1.3 Checklist on Equations and inequalities

- Formulate and solve a system of linear equations.
- Solve inequalities via algebraic method.
- Solve inequalities via graphical method.
- Solve a new inequality that looks similar to the previously solved one with a suitable substitution.

Syllabus topic 2: Sequences and series

2.1 Checklist on Summation

1. Sum of a, r, r^2, r^3, a^r and the like.
2. Break up the summation of a complex expression into a simpler one,
e.g. sum {2^r - 3r^2} = sum {2^2} - 3 sum {r^2}.
3. Using a summation result to deduce other results.

2.2 Checklist on Binomial expansion

1. Give the expansion in ascending/descending powers.
2. State validity range.
3. Use suitable substitution to find an approximate value.
4. Find coefficient of x^n term.

2.3 Checklist on AP/GP

1. Show series is AP/GP.
2. Solve a pure AP/GP problem (e.g. find term, sum, common difference, common ratio, sum to infinity).
3. Solve a problem where AP/GP are related (e.g. terms of AP are terms of GP).
4. Solve monetary problems (compound interest).
5. Solve problems involving patterns, e.g. find first term in the nth bracket given that {1}, {3, 5}, {7, 9, 11}, ...
6. Solve AP/GP problems described in the recurrence form, e.g. U_(n + 1) = d + U_n, U_(n + 1) = r(U_n).

2.4 Checklist on Method of differences

1. Use of partial fractions, trigonometric identities or appropriate algebraic manipulations to obtain difference of two similar expressions.
2. Cancellation of terms.
3. Find expression in terms of n or N.
4. Find the sum as n or N approaches infinity (concept of convergence/divergence).
5. Find an inequality for a summation of a similar form, based on the previous one.

2.5 Checklist on recurrences

1. Find the limit as n approaches infinity.
2. Prove some results or inequalities,
e.g. x_n > x_(n + 1).
3. Relate to graphs when proving, for instance,
x_n > x_(n + 1) when x_n < alpha.
4. Repeated use of the definition of recurrence to obtain a formula for U_n.
5. Formulate simple recurrence relations based on given problems.

2.6 Checklist on Mathematical induction

1. Prove a result that involves summation, then at times use the result to find sums.
2. Prove a result that involves recurrence.
3. Prove a conjecture, obtained by observing a pattern that comes from guided steps.

Syllabus topic 3: Vectors

- Use of ratio theorem and mid-point theorem to find position vectors and reflections of points and lines.
- Carry out scalar and vector product operations.
- Find lengths of projections (e.g. vector onto vector, vector onto line, vector onto plane, vector onto normal of plane).
- Find shortest distances (e.g. point & line, point & plane) with or without the need to find foot of perpendicular.
- Find areas (via vector product or standard formula like 1/2 bh).
- Find intersections (e.g. line & line, line & plane, plane & plane, 3 planes).

- Find angles (e.g. vector & vector, line & line, line & plane, plane & plane).
- Find equations of lines, planes, given information from the problem.
- Solve vector problems that involve given diagrams that describe some real-life situations.
- Give geometrical interpretations about line/plane relationships.

Syllabus topic 4: Complex numbers

- Find moduli and arguments of complex numbers, applying knowledge of z_1*z_2, z_1/z_2, (z)* and z^n.
- Convert between different forms i.e. cartesian, polar, exponential.
- Represent complex numbers as points on argand diagram and prove geometrical properties.
- Solve simple or simultaneous complex equations.
- Solve polynomial equations with real coefficients.
- Solve z^n = c (c is any complex number) type of equations.
- Factorise the equation based on roots found.
- Sketch loci (e.g. circles, perpendicular bisectors, half-lines) with or without inequalities.
- Find (via trigonometry or using GC to find intersections between loci) max/min moduli/arguments based on loci sketched.
- Find (via trigonometry or using GC to find intersections between loci) intersections of loci sketched.

Syllabus topic 5: Calculus

5.1 Checklist on Differentiation and its applications

- Differentiation techniques on various types of function.

- Equations of tangents and normals of curves defined in cartesian form, in parametric form or in implicit form:
1. find equations of tangents and normals;
2. find point where tangent/normal meets curve again;
3. find area formed by tangent/normal with the x- or y-axis;
4. find equations/points where tangent/normal is parallel to x-/y-axis.

- Rates of change (via chain rule).

- Maxima and minima:
1. Formulate an expression (usually area or volume), then find max/min.
2. Graph sketching with stationary points.

- Maclaurin's expansion:
1. Find expansion via standard series in formulae booklet.
2. Find expansion via repeated differentiation.
3. Small angle trigonometric approximations, which may involve the use of sine/cosine rule and/or trigonometric identities.
4. Use series to approximate values, to find definite integrals, to find limits, to find equation of tangent at the origin.
5. Solve an inequality involving the error between the actual function f and the Maclaurin's expansion of f.

5.2 Checklist on Integration and its applications

- Standard integration techniques.
- Integration by partial fractions.
- Integration by given substitutions.
- Integration by parts.
- Approximate areas via sum of rectangles, then find limit as number of rectangles approaches infinity.
- Find areas of curves defined parametrically.
- Find areas/volumes of curves defined in cartesian form or in implicit form.

5.3 Checklist on Differential equations

- Solve DE via direct integration.
- Solve DE via variable separable.
- Solve DE via substitution, followed by any of the above.
- Formulate DE based on a given problem description (some standard ones are listed below):
1. Newton's cooling model;
2. In-out rate flow model;
3. Birth-death model;
4. Money interest model.
Sometimes, the chain rule may be used.

- Find particular solutions.
- Sketch solution curves.
- Comment on appropriateness of model and/or interpret solutions (e.g. long-term behaviour).