Mastering Mathematics Smartly
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Further discussion

This is a nice question from a discussion on SgForums in which the graphical approach is superior to the algebraic approach of considering the discriminant.

Question:
The curve C is defined by the equation y = x/(x^2-5x+4).

Find a positive value for k such that the equation x/(x^2-5x+4) = kx has exactly 2 real roots.

Approach:
Sketch the graph of y = x/(x^2-5x+4).
One should realise that y = kx (where k > 0) will intersect y = x/(x^2-5x+4), provided that y = kx is tangent to the curve at the origin.
Find the value of dy/dx at the origin, which will be the value of k.

Using the algebraic method would only give k = -4/9.

This diagram shows that $y = \frac{1}{4}x$ and $y = -\frac{4}{9}x$ each intersects the curve twice.